



Research on the Multi-ships Cooperative Dilution of Precision Positioning Model in the Alert Water Environments

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Abstract: Currently, ship positioning mainly depends on the Global Navigation Satellite System (GNSS). Once the onboard navigation equipment malfunctions or the signal is artificially interfered with, the ship will be unable to position normally, and the navigation safety will also be threatened. To address the issue of how ships are positioned when the navigation system fails, a positioning model based on the Cooperative Dilution of Precision (CDOP) is proposed. This model expands the observables of the traditional satellite positioning system and adds the ranging information between ship nodes. Through the establishment of the CDOP model, the geometric configuration of the original ship navigation system is significantly improved. It has been demonstrated through theory and experiments that under the same measurement error conditions, the cooperative positioning accuracy of any user is superior to that of non-cooperative positioning, thereby enhancing the early warning and service information capabilities of the ship navigation system.

Keywords- Cooperative Dilution of Precision Positioning, Ship positioning, Geometric configuration

1. Introduction

Ship positioning^[1,2] refers to obtaining the position information and relevant data of ships in real-time through advanced technological means to ensure accurate navigation and safe operation during the voyage. With the development of globalization, the competition in the shipping industry has been intensifying, and ship positioning plays a vital role in shipping management and safety guarantee. Ship positioning systems mainly utilize technologies such as satellite navigation systems, radar, and radio communication to accurately acquire and track the positions of ships. According to the different positioning principles and technological applications, ship positioning can be classified into several types, such as global satellite navigation system positioning, local regional satellite navigation system positioning, radar positioning, and radio positioning. Global satellite navigation system positioning is one of the most common ship positioning methods at present, and representative systems include China's BD^[3], the United States' GPS^[4], Russia's GLONASS^[5], and the European Union's Galileo^[6], etc. This positioning method features global coverage, high precision, and high sensitivity, and can effectively meet the positioning requirements of ships during long-distance ocean voyages. Currently, China has accomplished the global networking construction of BD, and by 2035, will establish a more ubiquitous, more integrated, and more intelligent comprehensive spatial-temporal system.

However, once the onboard navigation equipment malfunctions or the signal is artificially interfered with, the ship will be unable to position normally, and the safety of navigation will be threatened. To improve the estimation accuracy of the ship's motion, in the past decade, scholars have carried out various research works on developing efficient multi-sensor fusion methods to estimate the motions and status of the ships^[7,8]. For instance, a model-driven discrete-time Kalman fusion, and a data-driven adaptive weighted fusion as the game objects estimating the ship's position was also proposed^[9]. A method combines multi-solid-state LiDAR point cloud and MMW Radar data to obtain berthing parameters of ship was proposed for assisting berthing^[10]. However, current

popular methods were proposed merely considering a single user as the fusion and expansion platform. Although expansion on a single and independent platform can improve the navigation performance of individuals, it still has numerous limitations in application scenarios. If a group of users is taken as the navigation service object, while applying the traditional multi-source fusion solution on each independent user, a cooperative positioning method for information sharing and relative position measurement among group users is added. When the observation quality is poor or the positioning conditions are not met, by obtaining the relative position relationship through cooperative users, users with degraded positioning performance or those unable to position independently can achieve a certain navigation accuracy, thus giving rise to the concept of cooperative positioning^[11-13].

In alert area positioning applications, the positioning and navigation accuracy of ships is significantly influenced by the environment, climate, and observation conditions. Introducing cooperative positioning into the ship positioning system and integrating it with the application of China's independently developed BDs can greatly improve the geometric structure configuration between users and satellites, as well as among users, thereby enhancing the positioning accuracy and improving the ship's early warning and service information capabilities. Based on this, this paper proposes a cooperative dilution of precision (CDOP) model and analyzes the validity of the proposed model and the positioning system from both theoretical and experimental aspects.

2. Cooperative Positioning System model

As depicted in Figure 1, in the warning water area, a cooperative positioning system is constituted by three user vessel nodes. The number of visible satellites for user vessel node #1 is 4. Due to signal occlusion, user node #2 cannot receive any satellite signals, and the number of visible satellites for user node #3 is 3. In the non-cooperative positioning mode, each user node independently calculates its position merely based on the satellite signals it receives. Hence, only user node #1 is capable of positioning its own location. Different from the non-cooperative positioning approach, in cooperative positioning, each user node, in addition to relying on satellite signals, also conducts relative distance measurement and communication with cooperative user nodes. After obtaining the relative position relationship with cooperative user nodes, the positions of all nodes in the area are jointly calculated.

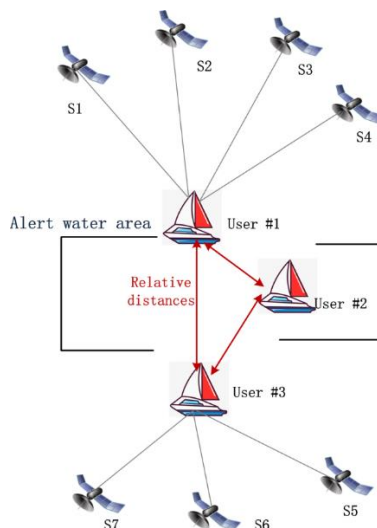


Figure 1. Cooperative Positioning System model in the Alert Water Environments

For any user n , the state is expressed as

$$\mathbf{x}_{u_n} = [x_{u_n}, y_{u_n}, z_{u_n}, \delta t_{u_n}], \quad n = 1, 2, \dots, N \quad (1)$$

Where $x_{u_n}, y_{u_n}, z_{u_n}$ represents the three-dimensional position vector of the user in the ECEF coordinate system, δt_{u_n} represents time offset.

The iterative solution of the cooperative positioning using the Gauss-Newton iteration at time k is

$$\mathbf{x}^k = \mathbf{x}^{k-1} + \Delta \mathbf{x}^{k-1} \quad (2)$$

$$\Delta \mathbf{x}^{k-1} = \left(\mathbf{G}^T (\mathbf{x}^{k-1}) \mathbf{W} \mathbf{G} (\mathbf{x}^{k-1}) \right)^{-1} \mathbf{v} (\mathbf{x}^{k-1}) \quad (3)$$

Where \mathbf{G} represent the Jacobian matrix, \mathbf{v} indicates measurement residual matrix, \mathbf{W} represents weight matrix.

3. Cooperative Dilution of Precision model

Let's assume that the relationship between the BD pseudorange error variance and the ranging error variance among users in the warning water area is expressed as

$$\frac{\gamma_r^2}{\gamma_\rho^2} = \zeta \quad (4)$$

Where γ_ρ^2 represents BD pseudorange error variance, γ_r^2 represents ranging error variance among users, ζ represents measurement error ratio.

The covariance matrix of BD cooperative positioning error $\hat{\delta}$ is computed as

$$\begin{aligned} \text{Cov}(\hat{\delta}) &= E(\hat{\delta} \cdot \hat{\delta}^T) \\ &= E\left((\mathbf{G}^T \mathbf{W} \mathbf{G})^{-1} \mathbf{G} \mathbf{W} \mathbf{v} \left((\mathbf{G}^T \mathbf{W} \mathbf{G})^{-1} \mathbf{G} \mathbf{W} \mathbf{v} \right)^T \right) \\ &= (\mathbf{G}^T \mathbf{Q} \mathbf{G})^{-1} \gamma_\rho^2 \end{aligned} \quad (5)$$

In formula (5), \mathbf{Q} is defined as

$$\mathbf{Q} = \mathbf{W} \gamma_\rho^2 = \begin{bmatrix} \mathbf{I}_K & \mathbf{0}_{K \times J} \\ \mathbf{0}_{J \times K} & \frac{\mathbf{I}_J}{\zeta} \end{bmatrix} \quad (6)$$

Where $\mathbf{0}_{J \times K}$ is a $J \times K$ zero matrix, \mathbf{I}_K represents a 2×2 unit matrix.

We define the cooperative weight coefficient matrix to be

$$\mathbf{H}_{\text{CP}} = (\mathbf{G}^T \mathbf{Q} \mathbf{G})^{-1} \quad (7)$$

Where the Jacobian matrix $\mathbf{G} = \begin{bmatrix} \mathbf{G}_\rho \\ \mathbf{G}_r \end{bmatrix}$ encompasses the number of team users, the number of visible satellites for each user, and the number of ranging links.

The diagonal elements of \mathbf{H}_{CP} indicate the variances of position and clock bias estimation errors, and the off-diagonal elements represent the correlation degrees among the estimated values, which is expanded as

$$\mathbf{H}_{CP} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_{1,2} & \cdots & \mathbf{H}_{1,N} \\ \mathbf{H}_{2,1} & \mathbf{H}_2 & \cdots & \mathbf{H}_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{N,1} & \mathbf{H}_{N,2} & \cdots & \mathbf{H}_N \end{bmatrix} \quad (8)$$

Where \mathbf{H}_n represents the cooperative weight coefficient matrix of user n , \mathbf{H}_{n_1, n_2} represents the cross-correlation matrix of user n_1 and n_2 .

Taking the user n as an example, the standard deviation of the spatial position positioning error can be calculated from \mathbf{H}_n as

$$\begin{aligned} \gamma_{P,n} &= \sqrt{\gamma_{E,n}^2 + \gamma_{N,n}^2 + \gamma_{U,n}^2} \\ &= \sqrt{h_{11,n} + h_{22,n} + h_{33,n}} \cdot \gamma_\rho \\ &= \text{PCDOP} \cdot \gamma_\rho \end{aligned} \quad (9)$$

Where $\gamma_{P,n}$ indicates standard deviation of spatial positioning error, $\gamma_{E,n}^2$, $\gamma_{N,n}^2$ and $\gamma_{U,n}^2$ represents the estimated variance of position in the east, north, and zenith directions respectively, $h_{ii,n}$ indicates the i th diagonal element of \mathbf{H}_n .

Spatial-Position Cooperative Dilution of Precision (PCDOP) is defined as

$$\text{PCDOP}_n = \sqrt{h_{11,n} + h_{22,n} + h_{33,n}} \quad (10)$$

Similarly, the geometric cooperative dilution of precision (GCDOP), horizontal cooperative dilution of precision (HCDOP), vertical cooperative dilution of precision (VCDOP), and time cooperative dilution of precision (TCDOP) are respectively defined as

$$\text{GCDOP}_n = \sqrt{h_{11,n} + h_{22,n} + h_{33,n} + h_{44,n}} \quad (11)$$

$$\text{HCDOP}_n = \sqrt{h_{11,n} + h_{22,n}} \quad (12)$$

$$\text{VCDOP}_n = \sqrt{h_{33,n}} \quad (13)$$

$$\text{TCDOP}_n = \sqrt{h_{44,n}} \quad (14)$$

Then, the standard deviation of the cooperative positioning errors for the user n is represented as

$$\gamma_{P,n} = \text{PCDOP} \cdot \gamma_{\rho} \quad (15) \quad \gamma_{G,n} = \text{GCDOP} \cdot \gamma_{\rho} \quad (16)$$

$$\gamma_{H,n} = \text{HCDOP} \cdot \gamma_{\rho} \quad (17) \quad \gamma_{V,n} = \text{VCDOP} \cdot \gamma_{\rho} \quad (18)$$

$$\gamma_{T,n} = \text{TCDOP} \cdot \gamma_{\rho} \quad (19)$$

Where $\gamma_{G,n}$ represents the standard deviation of geometric positioning error for the user n , $\gamma_{H,n}$, $\gamma_{V,n}$ and $\gamma_{T,n}$ represents the standard deviation of horizontal, vertical and time positioning error for the user n respectively.

Formulas (15) - (19) clearly indicate that the standard deviation of the pseudorange measurement error is magnified by the CDOP and transformed into the standard deviation of the cooperative positioning error in the corresponding direction. In terms of the defined model, under the same measurement error conditions, a smaller value of CDOP implies a potentially smaller cooperative positioning error. Each CDOP is calculated from the cooperative weight coefficient matrix, and the Jacobian matrix that determines the cooperative weighting matrix is associated with the geometric distribution of the visible satellites and effective ranging for each user. When the geometric distribution is favorable, the relative position distribution of each satellite with respect to the cooperative user is more uniform, and the corresponding CDOP value is smaller.

4. Comparison of cooperative and non-cooperative positioning accuracy models

Compared with non-cooperative positioning in BD, the cooperative positioning adds the measurement of the relative distance between ship users in the measurement system. When the number of visible satellites is small or insufficient for satellite solution, as an expansion of the observation equation, it participates in the joint solution of the positions, velocities and attitudes of the users. In this section, the positioning accuracy of the cooperative positioning method and the traditional independent positioning (or non-cooperative positioning) will be analyzed theoretically below.

If each user in the cooperative positioning team can achieve traditional independent positioning, then $\mathbf{G}_{\rho}^T \mathbf{G}_{\rho}$ is an invertible matrix, and its cooperative weight coefficient matrix \mathbf{H}_{CP} can be expanded as

$$\begin{aligned} \mathbf{H}_{\text{CP}} &= \left(\begin{bmatrix} \mathbf{G}_{\rho} \\ \mathbf{G}_r \end{bmatrix}^T \begin{bmatrix} \mathbf{I}_K & 0 \\ 0 & \frac{\mathbf{I}_J}{\zeta} \end{bmatrix} \begin{bmatrix} \mathbf{G}_{\rho} \\ \mathbf{G}_r \end{bmatrix} \right)^{-1} \\ &= \left(\mathbf{G}_{\rho}^T \mathbf{G}_{\rho} + \frac{\mathbf{G}_r^T \mathbf{G}_r}{\zeta} \right)^{-1} \end{aligned} \quad (20)$$

Where $\mathbf{G}_{\rho}^T \mathbf{G}_{\rho}$ and $\mathbf{G}_r^T \mathbf{G}_r$ are respectively related to pseudorange and relative range measurement observables, and both are symmetric positive semi-definite matrices. Then $\frac{\mathbf{G}_r^T \mathbf{G}_r}{\zeta}$ is a symmetric positive semi-definite matrix as well.

We define the matrix $\mathbf{J} = \mathbf{G}_{\rho}^T \mathbf{G}_{\rho}$, which is further expanded as

$$\begin{aligned}
 \mathbf{J} &= \begin{bmatrix} \mathbf{G}_{1,\rho}^T \mathbf{G}_{1,\rho} & 0 & \cdots & 0 \\ 0 & \mathbf{G}_{2,\rho}^T \mathbf{G}_{2,\rho} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \mathbf{G}_{N,\rho}^T \mathbf{G}_{N,\rho} \end{bmatrix} \\
 &= \begin{bmatrix} \mathbf{H}_{1,\rho}^{-1} & 0 & \cdots & 0 \\ 0 & \mathbf{H}_{2,\rho}^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \mathbf{H}_{N,\rho}^{-1} \end{bmatrix}
 \end{aligned} \quad (21)$$

Where $\mathbf{H}_{n,\rho}$ represents the weight coefficient matrix of traditional independent positioning for user n .

According to the Woodbury formula, the calculation of \mathbf{H}_{CP} is performed as

$$\begin{aligned}
 \mathbf{H}_{CP} &= \left(\mathbf{J} + \frac{\mathbf{G}_r^T \mathbf{G}_r}{\zeta} \right)^{-1} \\
 &= \mathbf{J}^{-1} - \mathbf{J}^{-1} \mathbf{G}_r^T (\zeta \mathbf{I}_J + \mathbf{G}_r \mathbf{J}^{-1} \mathbf{G}_r^T) \mathbf{G}_r \mathbf{J}^{-1}
 \end{aligned} \quad (22)$$

Since \mathbf{J}^{-1} is a symmetric positive semi-definite matrix, its eigenvalue decomposition is

$$\mathbf{J}^{-1} = \mathbf{U}_J^T \mathbf{A}_J \mathbf{U}_J \quad (23)$$

Where \mathbf{A}_J is a diagonal matrix, \mathbf{U}_J is an orthogonal matrix.

Express the matrix $\mathbf{U}_J \mathbf{G}_r^T$ in the form as follows

$$\mathbf{U}_J \mathbf{G}_r^T = \begin{bmatrix} j_1 \\ j_2 \\ \vdots \\ j_{4N} \end{bmatrix} \quad (24)$$

Multiply the left side of formula (24) by $\mathbf{G}_r \mathbf{U}_J^T \mathbf{A}_J$ and expand it as follows

$$\begin{aligned}
 \mathbf{G}_r \mathbf{U}_J^T \mathbf{A}_J \mathbf{U}_J \mathbf{G}_r^T &= \mathbf{G}_r \mathbf{J}^{-1} \mathbf{G}_r^T \\
 &= \sum_{i=1}^{4N} \lambda_{i,j} j_i^T j_i
 \end{aligned} \quad (25)$$

Since $\mathbf{G}_r \mathbf{J}^{-1} \mathbf{G}_r^T$ is a symmetric positive semi-definite matrix, the eigenvalue decomposition of $\zeta \mathbf{I}_J + \mathbf{G}_r \mathbf{J}^{-1} \mathbf{G}_r^T$ can be conducted as

$$\zeta \mathbf{I}_J + \mathbf{G}_r \mathbf{J}^{-1} \mathbf{G}_r^T = \mathbf{U}_1^T (\zeta \mathbf{I}_J + \mathbf{A}_1) \mathbf{U}_1 \quad (26)$$

Where \mathbf{A}_1 is a diagonal matrix, \mathbf{U}_1 is an orthogonal matrix.

By substituting formula (26) into formula (22), the following is obtained

$$\begin{aligned} \mathbf{H}_{CP} &= \mathbf{J}^{-1} - \mathbf{J}^{-1} \mathbf{G}_r^T \mathbf{U}_1^T (\zeta \mathbf{I}_J + \mathbf{A}_1)^{-1} \mathbf{U}_1 \mathbf{G}_r \mathbf{J}^{-1} \\ &= \mathbf{J}^{-1} - \mathbf{K}_1^T (\zeta \mathbf{I}_J + \mathbf{A}_1)^{-1} \mathbf{K}_1 \end{aligned} \quad (27)$$

Where $\mathbf{K}_1 = \mathbf{U}_1 \mathbf{G}_r \mathbf{J}^{-1}$.

Similar to formula (24), the matrix \mathbf{K}_1 can be represented as

$$\mathbf{K}_1 = \begin{bmatrix} k_{1,1} \\ k_{2,1} \\ \vdots \\ k_{J,1} \end{bmatrix} \quad (28)$$

Rewrite the matrix in formula (27) as $\mathbf{M}_1 = \mathbf{K}_1^T (\zeta \mathbf{I}_J + \mathbf{A}_1)^{-1} \mathbf{K}_1$, which is computationally equivalent to

$$\mathbf{M}_1 = \sum_{i=1}^J \left((\zeta + \lambda_{i,1})^{-1} k_{i,1}^T k_{i,1} \right) \quad (29)$$

Since both $\lambda_{i,1}$ and ζ are greater than zero, \mathbf{M}_1 remains a symmetric positive semi-definite matrix. At the same time, the matrix \mathbf{J}^{-1} is also a symmetric positive semi-definite matrix. As can be seen from formula (21), its nth diagonal matrix is the weight coefficient matrix $\mathbf{H}_{n,\rho}$ of the nth user's independent positioning. According to the non-negative property of a symmetric positive semi-definite matrix, the following inequality relationship can be deduced from formula (27)

$$\underbrace{h_{jj,n}}_{\mathbf{H}_{CP}} = \underbrace{h_{jj,n,\rho}}_{\mathbf{H}_{n,\rho}} - m_{1,jj,n} \leq h_{jj,n,\rho}, \quad 1 \leq j \leq 4 \quad (30)$$

Where $h_{jj,n,\rho}$ represents the jth diagonal element in matrix $\mathbf{H}_{n,\rho}$, the $[4(n-1)+j]$ th diagonal element in matrix \mathbf{M}_1 .

Take user n as an example. It can be known from the analysis in the previous subsection that the DOP value is determined by $\mathbf{H}_{n,\rho}$ which is the nth diagonal matrix of \mathbf{J}^{-1} , while the CDOP value is determined by the \mathbf{H}_{CP} . Therefore, based on formula (30), the following inequality relationship is deduced

$$\text{PCDOP}_n \leq \text{PDOP}_n \quad (31)$$

$$\text{GCDOP}_n \leq \text{GDOP}_n \quad (32)$$

$$\text{HCDOP}_n \leq \text{HDOP}_n \quad (33)$$

$$\text{VCDOP}_n \leq \text{VDOP}_n \quad (34)$$

$$TCDOP_n \leq TDOP_n \quad (35)$$

Thus, under the same pseudorange measurement error, the relationship between the error of cooperative positioning and that of on-cooperative positioning has always conformed to $\gamma_{CP} \leq \gamma_{GNSS}$. If a user in the team is unable to perform independent positioning, the DOP value cannot be calculated or is regarded as infinite. When the number of measurements in the cooperative positioning system is not less than the number of unknown parameters, the cooperative positioning method can be employed to solve the unknown parameters of the users and calculate the CDOP value. At this juncture, cooperative positioning can significantly enhance the positioning accuracy and availability of the current team's users.

In conclusion, through the derivation of the above formulas: For any user in the team, the CDOP value is always no greater than the DOP value which means that the positioning accuracy of cooperative positioning is always greater than or equal to that of traditional independent positioning under the same measurement error circumstances.

5. Analysis of Simulation Experiments

To validate the relationship between the CDOP model and the positioning performance of the cooperative positioning system, two experiments were designed on the precision of cooperative positioning. In the simulation experiment, three static users, namely users #1, #2 and #3, were designed. The satellite ephemerides were obtained from the original RINEX data output by the Trimble BD930 receiver. The relevant positioning solutions were accomplished in combination with the simulation data.

(1) The Comparison Experiment of Cooperative and Non-cooperative Accuracy

In the simulation experiments of this part, the visible satellites involved in the solution process for each user are completely the same. With the coordinate (34.0314N, 108.7635E, 422.28) in the geodetic coordinate system of user 1# as the coordinate origin, the simulation parameters such as the position information of the three users in station-centered coordinate systems, the visible satellites, and the measurement errors are presented in Table 1. The output frequencies of the pseudorange and ranging are both 1 Hz. The simulation generates 2000 epochs of observation data for cooperative and non-cooperative positioning solutions. During the experiment makes $\zeta = 6.25$.

Table 1 Position of Users in Station-Centered Coordinate System and Simulation Parameters

Simulation Parameters	User Coordinates (m)			Standard Deviation of Measurements (m)		The PRN number of visible satellites
	East-West	North	Up	Pseudo range	Ranging between users	
user #1	0	0	0	5	2	G05、G13、G15、
user #2	40	0	0	5	2	G18、G20、G21、
user #3	40	40	0	5	2	G24、G29

Figure 2 presents the comparison of cooperative positioning and non-cooperative positioning in various CDOP and DOP values. In the cooperative positioning system consisting of users #1, #2, and #3, the geometric, spatial, horizontal, vertical, and time cooperative dilution of precision of each user are all superior to the corresponding DOP values in the non-cooperative positioning system. In this experiment, as the measurement errors and visible satellites of each user are identical, the DOP values of all users are the same. The histogram of DOP values presented in Figure 2 can represent the results of all non-cooperative positioning users. Figure 3 presents the

comparison of RMSE in the east-north-up direction. It is shown that the RMSE of non-cooperative positioning in the east, north, and celestial directions is all larger than that of the user #1, #2 and #3 in the cooperative positioning system.

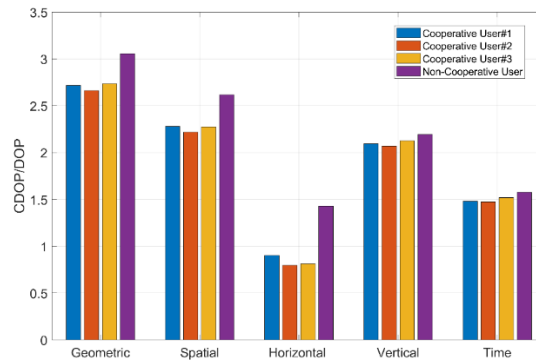


Figure 2 Comparison of CDOP/DOP Values between GNSS Cooperative Positioning and Non-cooperative Positioning

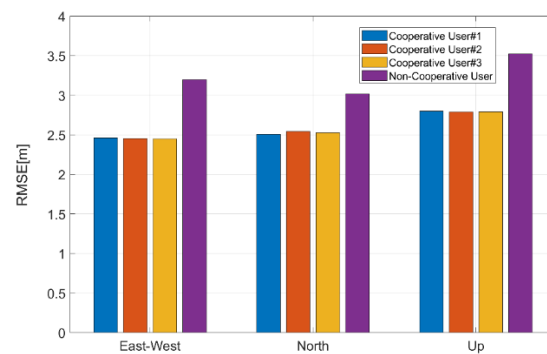


Figure 3 The Comparison of RMSE between Cooperative Positioning and Non-cooperative Positioning

(2) The influence of the measurement error ratio ζ on the accuracy of cooperative positioning

In this experiment, the relative positions of users #1, #2 and #3 remained as presented in Table 2, while the visible satellite were different from those in the previous experiment. The visible satellite scenarios and detailed simulation parameters for each user are shown in Table 2.

Table 2 Visible Satellites and Simulation Parameters

Simulation Parameters	Measurement Scenario		The PRN number of the visible satellite		
	Pseudorange Error (m)	Measurement Error ratio ζ	User #1	User #2	User #3
Values	5	$\lg 10^{-1} \leq \zeta \leq \lg 10^1$	G5,G15, G18,G20, G21,G24	G13,G18, G20,G21, G24,G29	G5,G15, G20,G21, G24,G29

The GCDOP/GDOP values of the three cooperative users in this simulation environment are depicted as shown in Figure 4. The curves of distinct colors indicate the GCDOP/GDOP values of users with varying satellite geometric distributions. For each cooperative user, with the increase of the ζ , the GCDOP shows an upward trend. Regardless of the changes in ζ , the GCDOP of the same user is not greater than its GDOP value, and with the increase of the ζ they eventually converge to the GDOP value. At present, the UWB ranging technology has been capable of guaranteeing $\zeta < 1$, which enables $GCDOP < GDOP$. It can be further observed that the

measurement error ratio ζ has a minor influence on the cooperative positioning accuracy. In other words, high-precision ranging technology is not a requisite for achieving cooperative positioning and its applications.

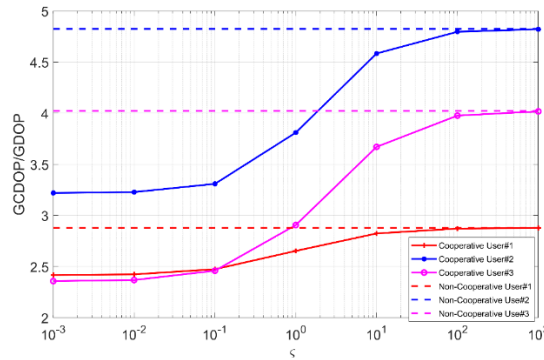


Figure 4 The variation of GCDOP along with ζ

At present, the UWB ranging technology is already capable of guaranteeing $\zeta < 1$, which enables $GCDOP < GDOP$. It can be further discerned that the measurement error factor ζ has a relatively minor influence on the cooperative positioning accuracy. In other words, high-precision ranging technology is not a requisite for achieving cooperative positioning and its applications.

6. Conclusion

In summary, this paper has designed two experiments to verify the positioning performance of a cooperative positioning system and compared it with non-cooperative positioning system. The CDOP of all cooperative positioning parameters is better than the DOP value of non-cooperative positioning system, which proves that the positioning accuracy of any user cooperative positioning is better than non-cooperative positioning under the same measurement error. Additionally, the experiments also verify the correctness and effectiveness of the cooperative positioning model.

Funding: This work is supported by National Natural Science Foundation of China (Grant Number 62341119), Natural Science Foundation of Jiangsu Province for Youth (Grant Number BK20210941), Changzhou Leading Innovative Talent Introduction and Cultivation Project (Grant Number CQ20210094).

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