

An Approach of Linear Programming: Optimal Irrigation Time for Water Management

Garima Sharma^{1a}, Arun Kumar Chaudhary^{1b}, Lal Babu Sah Telee², Murari Karki³, Bidur Nepal⁴,
Kameshwar Sahani⁵, Suresh Kumar Sahani^{*6}

^{1a}Department of Mathematics, Mody University of Science and Technology, Sikar, India.

sharmagarima2802@gmail.com

^{1b}Department of Statistics, Nepal Commerce Campus, T.U., Nepal

akchaudhary1@yahoo.com

²Department of Statistics, Nepal Commerce Campus, T.U., Nepal

lalbabu3131@gmail.com

³Department of Statistics, Saraswati Multiple Campus, T.U., Nepal

murari.karki@smc.tu.edu.np

⁴Department of Statistics, Patan Multiple Campus, T.U., Nepal

bnepalpatan027@gmail.com

⁵Department of Civil Engineering, K.U., Nepal

kameshwar.sahani@ku.edu.np

^{*6}Faculty of Science, Technology, and Engineering, R.J.U., Nepal

sureshsahani@rju.edu.np

Corresponding Authors: sureshsahani@rju.edu.np

sharmagarima2802@gmail.com

kameshwar.sahani@rju.edu.np

lalbabu3131@gmail.com

Abstract: This study delves into minimizing irrigation time to safeguard water resources although maintaining crop health and yield. As the world's population is increasing at a rapid rate, there is an increased pressure on water resources and to fulfil the demand of current and future generation, we must adopt water management in agriculture ensuring food security, economic and environmental stability, water conservation and social equity. For this, we optimize irrigation time to conserve water, prevent soil erosion and reduce energy usage resulting in more sustainable agricultural practices. A Linear Programming model is formulated considering the constraints as water availability, land availability, irrigation system capacity.

Keywords- Optimization, LPP formulation, Irrigation, Simplex method, Soil type

1. Introduction

Farmers must choose which crops to sow in each of their parcels at the start of the production cycle. Because it is influenced by the geographic diversity of the physical and chemical soil qualities inside each parcel, it is one of the most complicated decisions. Because soil variability has a significant impact on water balance, nutrient dynamics, and responsiveness to input application (seeds and fertilizers), it directly influences crop pattern choice. For example, growing maize or tomatoes on a plot that is heavy in phosphate and nitrogen could increase yields without requiring a lot of fertilizer. However, the farmers may choose to fertilize only half of the parcel rather than the entire parcel if half of it has low nitrogen levels. Furthermore, crop planning decisions must take into

account a number of factors, including the expected prices of crops yielded, the expected amount of available water for the production cycle, the cost of irrigating a parcel (some parcels may be remote, and irrigation may require more electricity), the phenological stages of the crops, the number of hectares in each parcel, the expected amount of resources, and so on [see, 1].

Linear Programming also acknowledged as linear optimization is defined as a procedure for reaching the finest achievable result in a mathematical model expressed in linear relationship. In its core, Linear Programming is akin to a puzzle in which the limited pieces should be kept in order for receiving the complete picture. The term “Linear Programming” includes two terms: Linear and Programming. The term “Linear” states that in a problem, relations amid two or more variables is linear. The term “Programming” represents planning.

At the time of scarcity such as Second World War, linear programming plays a crucial role in military logistics and operations. By creating mathematical model for allocation of resources like fuel, ammunition and manpower, impactful decisions were made by military planners which impacted outcomes of battles and campaigns.

In 2014, Tukimat et.al introduced ideas for Pedu-Muda reservoir operation policy to enhance the reservoir operation to fit into impactful climate change [see, 2]. In 2016, Singh et.al proposed a theory highlighting different optimization tools for planning and managing the integrated use of water resources in irrigated agriculture [see, 3]. In 2017, Difallah et.al proposed a study in which a linear programming model is formulated to optimize water usage blended with innovative irrigation techniques considering precipitation effectiveness to determine the required amount of irrigation water [see, 4]. In 2020, Bhatia et.al analyzed cropping patterns in different districts of Rajasthan to boost farm outputs and revenue [see, 5]. In 2021, Imron addressed the issue of Belitang Irrigation System. The study aims to optimize irrigation water allocation ensuring maximal benefits [see, 6]. In 2022, Zhang et.al highlighted the major issue of drought effecting crop growth impacting crop yield and water use efficiency. This study focuses on optimizing irrigation time and water to conserve water [see, 7]. In 2024, Sejati et.al proposed a theory optimizing irrigation water availability in Klakah irrigation area, Lumajang Regency, maximizing the agricultural profit and production ensuring irrigation efficiency [see, 8].

2. Objectives

In our problem, we ought to minimize the irrigation time for Wheat, Mustard and Cotton in Mahendragarh, Haryana ensuring that the crop water needs are fulfilled and the available land is utilized efficiently by an approach of Linear Programming Problem. First we convert the problem into LPP model and after that we solve the problem by simplex method method for conclude the optimal solution.

3. Methods

Let us consider a problem with q decision variables, i.e. $y_1, y_2, y_3, \dots, y_q$

Optimize (Minimum or Maximum) $Z = c_1y_1 + c_2y_2 + c_3y_3 + \dots + c_qy_q$

Subject to constraints:

$$u_{11}y_1 + u_{12}y_2 + \dots + u_{1q}y_q (\leq \text{Or} = \text{or} \geq) v_1$$

$$u_{21}y_1 + u_{22}y_2 + \dots + u_{2q}y_q (\leq \text{Or} = \text{or} \geq) v_2$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$u_{p1}y_1 + u_{p2}y_2 + \dots + u_{pq}y_q (\leq \text{Or} = \text{or} \geq) v_p$$

Where, $y_1, y_2, \dots, y_q \geq 0$

Compact form of LPP is expressed as:

Optimize (Max or Min) $Z = \sum_{j=1}^q c_j y_j$

Subject to constraints:

$$\sum_{j=1}^q u_{ij} y_j (\leq \text{Or } = \text{or } \geq) v_i$$

Where, $y_j \geq 0$ ($j = 1, 2, \dots, q$)

And, c_j = Cost coefficient ($j = 1, 2, \dots, q$)

And, u_{ij} = Substitution coefficient ($i = 1, 2, \dots, p$) and ($j = 1, 2, \dots, q$)

And, v_i = Requirements ($i = 1, 2, \dots, p$)

Even though the planet Earth is indeed abundant in water, only a bare minimum of it that is just one percent is useful to sustain life. As per the World Bank report, 40 % of world's population is affected by water-scarcity and 70% of deaths are caused by water disasters. Despite of current condition of water resources, people are wasting a lot of water and this was even observed by me in my hometown Mahendragarh, Haryana where agricultural provides livelihood to a lot of people. But still water is wasted in huge amount while irrigating crops.

The crops considered for our research are Wheat, Mustard and Cotton as these are the most sown crops there. Underneath is the data for my problem which have been collected from different resources:

Crops	Water Requirement(mm/ha)	Irrigation System Capacity(hrs/ha)	Land Availability(ha)
Wheat(y_1)	363.4	1	43600
Mustard(y_2)	403	1	7600
Cotton(y_3)	900	1	4000
Total Availability	11648 mm/ha	73.87 hrs/ha	121000 ha

We ought to minimize the irrigation time for Wheat, Mustard and Cotton in Mahendragarh ensuring that the crop water needs are fulfilled and the available land is utilized efficiently.

The mathematical formulation for the problem is as follows:

$$\text{Min } Z = y_1 + y_2 + y_3$$

Subject to constraints:

$$363.4y_1 + 403y_2 + 900y_3 \leq 11648$$

$$y_1 + y_2 + y_3 \leq 73.87$$

$$43600y_1 + 7600y_2 + 4000y_3 \leq 121000$$

And non-negative restrictions i.e. $y_1, y_2, y_3 \geq 0$

Solution:

Simplex Method is applied to solve the linear programming problem.

The constraints in the problem consist of less than inequality. Adding slack variables to convert the inequalities into equalities and then applying simplex method to get optimal solution i.e.

$$\text{Min } Z = y_1 + y_2 + y_3 + 0S_1 + 0S_2 + 0S_3$$

Subject to constraints:

$$363.4y_1 + 403y_2 + 900y_3 + S_1 + 0S_2 + 0S_3 = 11648$$

$$y_1 + y_2 + y_3 + 0S_1 + S_2 + 0S_3 = 73.87$$

$$109y_1 + 19y_2 + 10y_3 + 0S_1 + 0S_2 + S_3 = 302.5$$

And $y_1, y_2, y_3 \geq 0$

Proceeding further to obtain optimal solution:

	C_j	1	1	1	0	0	0		
$B.V.$	C_B	y_1	y_2	y_3	S_1	S_2	S_3	X_B	Min.Ratio X_B/Y_K
S_1	0	363.4	403	900	1	0	0	11648	32.05
S_2	0	1	1	1	0	1	0	73.87	73.87
S_3	0	109	19	10	0	0	1	302.5	2.775
$Z_j - C_j = \Delta_j$		-1	-1	-1	0	0	0		

The most negative value of Δ_j is -1 so, we take incoming vector is y_1 and minimum ratio is 2.775 so outgoing vector is S_3 .

	C_j	1	1	1	0	0	0		
$B.V.$	C_B	y_1	y_2	y_3	S_1	S_2	S_3	X_B	Min.Ratio X_B/Y_K
S_1	0	0	341.2	866.676	1	0	-3.34	10639.565	12.276
S_2	0	0	0.83	0.908	0	1	-0.0092	71.095	78.27
y_1	1	1	0.17	0.0917	0	0	0.0092	2.775	30.26
$Z_j - C_j = \Delta_j$		0	-0.83	-0.908	0	0	0.0092		

The most negative value of Δ_j is -0.908 so incoming vector is y_3 and minimum ratio is 12.276 so outgoing vector is S_1 .

	C_j	1	1	1	0	0	0		
$B.V.$	C_B	y_1	y_2	y_3	S_1	S_2	S_3	X_B	Min.Ratio X_B/Y_K
y_3	1	0	0.394	1	0.00115	0	-0.00385	12.276	31.18
S_2	0	0	0.472	0	-0.001	1	-0.0057	59.945	126.89
y_1	1	1	0.134	0	-0.0001	0	0.00955	1.649	12.317
$Z_j - C_j = \Delta_j$		0	-0.4724	0	0.00105	0	0.0057		

The most negative value of Δ_j is -0.4724 so incoming vector is y_2 and minimum ratio is 12.317 so outgoing vector is y_1 .

	C_j	1	1	1	0	0	0		
$B.V.$	C_B	y_1	y_2	y_3	S_1	S_2	S_3	X_B	Min.Ratio X_B/Y_K
y_3	1	-2.94	0	1	0.001445	0	-0.0319	7.427	-
S_2	0	-3.528	0	0	-0.000646	1	-0.0394	54.126	-
y_2	1	7.468	1	0	-0.00075	0	0.0713	12.317	-
$Z_j - C_j = \Delta_j$		3.528	0	0	0.0007	0	0.0394		

In the last table all $\Delta_j's \geq 0$.

We can conclude that $y_2 = 12.317, y_3 = 7.427, S_2 = 54.126$ and rest of decision variables are zero.

And $\text{Min } Z = 12.317 + 7.427$

i.e., $\text{Min } Z = 19.744 \text{ hrs/ha}$

4. Results

The optimal solution i.e. the minimized irrigation time is 19.744 hrs/ha which can vary through changing conditions.

5. Discussion

It's not just about numbers; it's about the farmers who seek their livelihood in agriculture. This study is all about minimizing irrigation time so that the land is used efficiently and water wastage is minimized while ensuring that the crops receive adequate amount of water. The data considered in this study is gathered through the prior researches on crops supplemented with information through agricultural contingency plan of Mahendragarh. The data may vary according to the conditions such as; climatic condition, soil moisture, water availability.

But for managing water resources this idea can be taken into consideration. The optimal solution i.e. the minimized irrigation time is 19.744 hrs/ha which can vary through changing conditions.

As in the forecoming years i.e. by 2050, 50% of agricultural production will be required for feeding the population leading to an increase in 15% of water withdrawal. To fulfil the demand, we will require water management in agriculture ensuring food security, economic and environmental stability, water conservation and social equity. Only water is not conserved by this but it will also enhance crop growth even at the time of water scarcity.

Recommended Techniques to Maximize Water Productivity and Reduce Wastage:

- Efficient Irrigation Techniques such as drip irrigation, sprinkler irrigation and micro-irrigation systems.
- The execution of methods such as sustainable agriculture, combination of different pest control techniques and agroforestry can enhance soil health and reduce irrigation water and pest needs.
- Retaining of soil moisture can minimize water need ensuring proper crop growth mainly in arid and semi-arid regions.
- Crop rotation, planting of drought tolerant crops and intercropping systems can optimize water productivity during water deficiency conditions.
- The investment in infrastructure such as canals, reservoirs, ponds, tanks can store rainwater which can be utilized during dry period.

References

- [1] Cid-Garcia, N. M., Bravo-Lozano, A. G., & Rios-Solis, Y. A. (2014). A crop planning and real-time irrigation method based on site-specific management zones and linear programming. *Computers and electronics in agriculture*, 107, 20-28.
- [2] Tukimat, N. N. A., & Harun, S. (2014). Optimization of water supply reservoir in the framework of climate variation. *International Journal of Software Engineering and Its Applications*, 8(3), 361-378.
- [3] Singh, A., Panda, S. N., Saxena, C. K., Verma, C. L., Uzokwe, V. N., Krause, P., & Gupta, S. K. (2016). Optimization modeling for conjunctive use planning of surface water and groundwater for irrigation. *Journal of Irrigation and Drainage Engineering*, 142(3), 04015060.
- [4] Difallah, W., Benahmed, K., Draoui, B., & Bounaama, F. (2017). Linear optimization model for efficient use of irrigation water. *International Journal of Agronomy*, 2017.
- [5] Bhatia, M., & Rana, A. (2020). A mathematical approach to optimize crop allocation—A linear programming model. *Int. J. Des. Nat. Ecodynamics*, 15(2), 245-252.



- [6] Imron, F. (2021, February). Optimization of irrigation water allocation by using linear programming: case study on Belitang irrigation system. In IOP Conference Series: Earth and Environmental Science (Vol. 653, No. 1, p. 012023). IOP Publishing.
- [7] Zhang, X., Guo, P., Guo, S., Zhang, F., & Zhang, C. (2022). An integrated model to optimize irrigation amount and time in shallow groundwater area under drought conditions. *Journal of Contaminant Hydrology*, 246, 103956.
- [8] Sejati, W., & Akbar, T. T. (2024). Optimization Study of Cropping Pattern in the Klakah Irrigation Area, Lumajang Regency, Using Linear Programming. *ADI Journal on Recent Innovation*, 5(2), 136-145.